TM 5 PR 3.10

THE AMPLITUDE OF A DAMPED OSCILLATOR DECREASES TO $\frac{1}{e}A_0$ AFTER n PERIODS. SHOW

$$\frac{\omega_s}{\omega_N} \approx 1 - \frac{1}{8\pi^2 n^2}$$

USING THE DECREMENT OF MOTION

$$\frac{A(t_0+n\tau)}{A(t_0)} = \frac{1}{e} = \frac{A_0 e^{-\beta(t_0+n\tau)}}{A_0 e^{-\beta t_0}} = e^{-\beta n \tau}$$

THUS

$$1 = \beta n \tau$$

$$\Rightarrow \beta = \frac{1}{n\tau} = \frac{\omega_s}{2\pi n}$$

WRITING OUT ω_s,

$$\omega_s^2 = \omega_N^2 - \beta^2 = \omega_N^2 - \frac{\omega_s^2}{4\pi^2 n^2}$$

$$\omega_s^2\left(1 + \frac{1}{4\pi^2 n^2}\right) = \omega_N^2$$

THUS

$$\frac{\omega_s}{\omega_N} = \left(1 + \frac{1}{4\pi^2 n^2}\right)^{-\frac{1}{2}}$$

EXPANDING THE SQUARE ROOT, KEEPING 2 TERMS

$$\boxed{\frac{\omega_s}{\omega_N} \approx \left(1 - \frac{1}{8\pi^2 n^2}\right)} \quad \text{QED!}$$

NOTE BOOK